5

Two-way Comparisons -- Elementary Analysis

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5

Two-way Comparisons -- Elementary Analysis

Organized comparisons are not restricted to comparisons among a single set of candidates or treatments. Including two or more sets of treatments or candidates in a single experiment or a single data gathering is often advantageous and sometimes essential.

It usually turns out that the more structure in the data, the more easily and thoroughly we can look into it. Accordingly, we gain by having good ways of starting to look into highly structured data, ways that take full advantage of its structure.

When we have two sets of candidates, we naturally lay out the data in rows and columns. We are now going to treat rows and columns on an equal footing. One minor consequence is a need for a simple word - a noun-adjective - meaning either "row" or "column". The simplest choice seems to be "stripe" . Accordingly, our two-way tables are divided into stripes in two ways, once horizontally and once vertically.

5A. Symmetric analysis

In 1938, H. E. Daniels published a paper on Some Problems of Statistical Interest in Wool Research. One of his first examples dealt with the irregularity in weight of the threads, known as slubbing, formed in the process called carding, which is the only major preliminary to the spinning of wool. In the type of carding operation studied by Daniels, 100 threads

are produced, side by side, 25 being wound on each of four condenser bobbins. The 25 "ends" on bobbin 1, here numbered 1 to 25, come from positions 1,5,9,...,97, those on bobbin 2, also here numbered 1 to 25, come from positions 2,6,10,...,98, and so on.

Table 1 sets out, in condensed form, and partially analyzes the results of one experiment,

The only novel feature of this analysis is its last step. The bobbin adjustments are calculated to give us a compact way of stating the apparent bobbin differences. Since only differences are important, we are free to add the same constant, positive or negative, to all adjustments. In choosing what to do, we ought to be mainly concerned with clarity of result. If we had to put the differences among the adjustments found in 1(f) into words, we would probably say something like:

- bobbins 2 and 4 appear the same.
- bobbin 1 appears about 1.2 (gram per five ends) lighter.
- bobbin 3 appears about 0.2 (gram per five ends) heavier.

It would be nice for our final adjustments to be such as to bring out these simple words as naturally as possible.

The square combining table forces the sum -- and hence the mean -- of its adjustments to be zero. Fuller treatment disturbs this somewhat. To what standard should we readjust? Making the midmean zero -- easily done by subtracting

table 1 of chapter 5

Daniels's Weights of Slubbing and a Partial Analysis

a) WEIGHTS OF 5 ENDS, EACH 95 YARDS LONG, IN GRAMS

Ends	Bobbin number 1	2	3	4
1-5	38.43	38.83	39.08	39.35
6-10	40.27	41.63	41.28	41.19
11-15	40.42	42.18	41.42	42.28
16-20	40.22	42.05	41.47	41.95
21-25	40.27	40.22	41.18	40.99

b) COMPARISONS AMONG BOBBINS, * = MIDMEAN

	1-2	1-3	1-4	2-3	3-4	2-4
	-.40	-.65	-.92	-.25	-.27	-.52
	-1.36	-1.01	-.92	.35	.09	.44
	-1.76	-1.00	-1.86	.76	-.86	-.10
	-1.83	-1.25	-1.73	.58	-.48	.10
	0.05	-.91	-.72	-.96	.19	-.77
*	-1.19	-.98	-1.16	+.24	-.22	-.17

c) SQUARE COMBINING TABLE

Minus Bobbin	Plus Bobbin 1	2	3	4
1	0	1.19	.98	1.16
2	-1.19	0	-.24	.17
3	-.98	+.24	0	.22
4	-1.16	-.17	-.22	0
Sums	-3.33	1.26	.52	1.55
Means	-.83	.32	.13	.39

d) FITTED DIFFERENCES

1	0	1.15	+.96	1.22
2	-1.15	0	-.19	.07
3	-.96	.19	0	.26
4	-1.22	-.07	-.26	0

table 1 (Cont'd) - 2

e) COMBINING RESIDUALS, * = MIDMEAN

	1	2	3	4
1	0	.04	.02	-.06
2	-.04	0	-.05	.10
3	-.02	.05	0	-.04
4	+.06	-.10	.04	0
*	-.01	+.02	+.01	-.02

f) FULLER TREATMENT ADJUSTMENTS = MIDMEAN POLISHED ADJUSTMENTS

means (c)	-.83	.32	.13	.39	
* (e)	-.01	.02	+.01	-.02	
	-.84	.34	.14	.37	midmean = .24

g) REVISED ADJUSTMENTS = ADJUSTMENT - MIDMEAN

-1.08 .10 -.10 .13

their midmean from all the adjustments -- seems to be as good a choice as we now know how to make.

Accordingly, panel (g) gives the revised adjustments. For this set of data they do exceptionally well in making the simple words also natural.

This analysis is clearly only part of what can -- and should -- be done with this data. We have compared the four bobbins. The five groups of ends are equally deserving of comparison. Table 2 turns the data table around -- for convenience, not of necessity -- and makes the analysis among the five groups.

Now we have looked carefully at:

- comparisons among bobbins,
- comparisons among end groups,

we are ready to use the adjustments thus found as a step toward

- formation of individual residuals.
- looking deeper into the data.

Let us begin in the upper left corner of table 1. We find there

38.43 for bobbin 1 and group A

the relevant adjustments are:

-1.08 for bobbin 1

-2.10 for group A

and the result of applying them is

$$38.43 - (-1.08) - (-2.10) = 41.61$$

table 2 of chapter 5

The Same Data Further Analyzed

a) WEIGHTS OF 5 ENDS, EACH 95 YARDS LONG

Bobbin	Ends (in groups of five) A (1-5)	B (6-10)	C (11-15)	D (16-20)	E (21-25)
1	38.43	40.27	40.42	40.22	40.27
2	38.83	41.63	42.18	42.05	40.22
3	39.08	41.28	41.42	41.47	41.18
4	39.35	41.19	42.28	41.95	40.99

b) DIRECT COMPARISONS AMONG GROUPS, * = MIDMEANS

	A-B	A-C	A-D	A-E	B-C	B-D	B-E	C-D	C-E	D-E
	-1.84	-1.99	-1.79	-1.84	-.15	.05	.00	.20	.15	-.05
	-2.80	-3.35	-3.22	-1.39	-.55	-.42	1.41	.13	1.96	1.83
	-2.20	-2.34	-2.39	-2.10	-.14	-.19	.10	-.05	.24	.29
	-1.84	-2.93	-2.60	-1.64	-1.09	-.76	.20	.33	1.29	.96
*	-2.02	-2.64	-2.50	-1.74	-.35	-.30	.15	.16	.76	.62

c) SQUARE COMBINING TABLE

Minus group	Plus group A	B	C	D	E
A	0	2.02	2.64	2.50	1.74
B	-2.02	0	.35	.30	-.15
C	-2.64	-.35	0	-.16	-.76
D	-2.50	-.30	.16	0	-.62
E	-1.74	.15	.76	.62	0
Sum	-8.90	1.52	3.91	3.26	.21
Mean	-1.78	.30	.78	.65	.04

d) FITTED DIFFERENCES

A	0	2.08	2.56	2.43	1.82
B	-2.08	0	.48	.35	-.26
C	-2.56	-.48	0	.13	-.74
D	-2.43	-.35	.13	0	-.61
E	-1.82	.26	.74	.61	0

table 2 (Cont'd) - 2

	A	B	C	D	E
e) COMBINING RESIDUALS, * = MIDMEAN					
A	0	-.06	.08	.07	-.08
B	.06	0	-.13	-.05	.11
C	-.08	.13	0	-.03	-.02
D	-.07	.05	.03	0	-.01
E	.08	-.11	.02	.01	0
*	.00	.00	.02	-.01	-.01
f) FINAL ADJUSTMENTS					
means (c)	-1.78	.30	.78	.65	.04
* (e)	.00	.00	.02	-.01	-.01
sums	-1.78	.30	.80	.64	.03 (.32)
final	-2.10	-.02	.48	.32	-.29

The result of doing this throughout the data is shown in panel (a) of table 3. These 20 adjusted values are looked at rather generally in panel (b), and their midmean found to be 41.34. This midmean is then subtracted to give the data residuals shown in (c). The first two columns show a tendency to a bow-shaped structure -- one one way, one the other -- but the other columns do not seem to reinforce this appearance.

5B. Elementary analysis

While one must crawl before one leaps, it is sometimes important to leap. In the last section we went carefully -- step by step -- through the analysis of the 20 yarn weights. With luck, all the steps are clear -- but how can there yet be clarity about the overall situation, which we have not looked at?

Table 4 summarizes the data and the results in a pattern to which we will come back again and again. The process is simple to say:

- we took apart the original table into four components which ADD up to the original values; namely

1) a general typical, which indicates the general level of the values,

2) row adjustments, which indicate the apparent deviation of each row from this overall level,

3) column adjustments, which indicate the apparent deviation of each column from the same overall level,

4) residuals, which indicate the apparent deviation of each individual entry from the partial description provided by the sum of the other three constituents.

table 3 of chapter 5

Adjusted Values of Various Sorts
for Daniels's Thread Weights

a) ORIGINAL WEIGHTS ADJUSTED FOR THE FINAL ADJUSTMENTS OF TABLES 1 AND 2

	Bobbin			
Group*	1	2	3	4
A	41.61	40.83	41.28	41.32
B	41.37	41.55	41.40	41.08
C	41.02	41.60	41.04	41.67
D	40.98	41.63	41.25	41.50
E	41.64	40.41	41.57	41.15

b) STEM-AND-LEAF LAYOUT OF SAME

```
41.7 |
   6 | 10734
   5 | 507
   4 | 0
   3 | 27
   2 | 85
   1 | 5
   0 | 824
40.9 | 8
   8 | 3
   7 |
   6 |
   5 |
   4 | 1
```

hinges: 41.06, 41.58
midmean: 41.34 (actually 41.35!)

40.41 is far out (beyond 40.54)

c) ENTRIES OF (a) FURTHER ADJUSTED BY THEIR MIDMEAN

	1	2	3	4
A	.27	-.51	-.06	-.02
B	.03	.21	.06	-.26
C	-.32	.26	-.30	.33
D	-.36	.29	-.09	.16
E	.30	-.93**	.23	-.19

* = of ends.
** = far out.

table 4 of chapter 5

The Elementary Analysis of the 20 Thread Weights

a) THE DATA

Ends	Bobbin 1	2	3	4
A	38.43	38.83	39.08	39.35
B	40.27	41.63	41.28	41.19
C	40.42	42.18	41.42	42.28
D	40.22	42.05	41.47	41.95
E	40.27	40.22	41.18	40.99

b) THE PATTERN OF THE ELEMENTARY ANALYSIS

	tags	⋮ --
tags	residuals	*
... -	**	***

*One column of row adjustments. - Kind of column tags.

**One row of column adjustments. -- Kind of row tags.

***One general typical value.

c) THE ELEMENTARY ANALYSIS

	1	2	3	4	⋮ Bobbin
A	.27	-.51	-.06	-.02	-2.10
B	.03	.21	.06	-.26	-.02
C	-.32	.26	-.30	.33	.48
D	-.36	.29	-.09	.16	.32
E	.30	-.93	.23	-.19	-.29
... Ends	-1.08	.10	-.10	.13	41.34

d) EXAMPLES OF IMPLIED REASSEMBLY

$$(.27) + (-2.10) + (-1.08) + (41.34) = 38.43$$
$$(-.51) + (-2.10) + (0.10) + (41.34) = 38.83$$
$$(.03) + (-.02) + (-1.08) + (41.34) = 40.27$$
$$(.21) + (-.02) + (0.10) + (41.34) = 41.63$$

We have made an elementary analysis in one way. It is a good way, but other ways can -- and do -- produce elementary analyses. Their results need not be the same as those just obtained.

When dealing with a single batch, we thought about residuals from means, from medians, from midmeans. We did not expect the residuals to be the same when the partial descriptions from which they are calculated are not the same. We found nothing strange in there being more than one possible partial description. Mean or median -- or midmean -- each seemed likely to have its place, each generated its own set of residuals. The essential was the nature of our partial description -- there a single number equally related to all the observations of the batch.

Here, again, the essential is the nature of our partial description, now:

- the SUM of three terms.
- one term the same everywhere.
- one term the same throughout each row.
- one term the same throughout each column.

Once we have chosen our partial description -- usually we do this by choosing the process that fixes it -- the residuals follow, in accordance with the general formula:

observation = (partial description) + (residual)

Sometimes we get a better picture of what is happening by expanding the constituents of the elementary analysis, as is done for our example in table 5. Here each panel consists

table 5 of chapter 5

The Elementary Analysis Expanded;
Data and Analysis of tables 1 to 4

	Bobbin			
Ends*	1	2	3	4
a) THE RESIDUALS				
A	.27	-.51	-.06	-.02
B	.03	.21	.06	-.26
C	-.32	.26	-.30	.33
D	-.36	.29	-.09	.16
E	.30	-.93	.23	-.19
b) THE ENDS (OR ROW) ADJUSTMENTS				
A	-2.10	-2.10	-2.10	-2.10
B	-.02	-.02	-.02	-.02
C	.48	.48	.48	.48
D	.32	.32	.32	.32
E	-.29	-.29	-.29	-.29
c) THE BOBBIN (OR COLUMN) ADJUSTMENTS				
A	-1.08	.10	-.10	.13
B	-1.08	.10	-.10	.13
C	-1.08	.10	-.10	.13
D	-1.08	.10	-.10	.13
E	-1.08	.10	-.10	.13
d) THE GENERAL TYPICAL				
A	41.34	41.34	41.34	41.34
B	41.34	41.34	41.34	41.34
C	41.34	41.34	41.34	41.34
D	41.34	41.34	41.34	41.34
E	41.34	41.34	41.34	41.34

*In groups of 5.

table 5 (Cont'd) - 2

e) THE FITTED PARTIAL DESCRIPTION**

Ends*	1	2	3	4
A	38.16	39.34	39.14	39.37
B	40.24	41.42	41.22	41.45
C	40.74	41.92	41.72	41.95
D	40.58	41.76	41.56	41.79
E	39.97	41.15	40.95	41.18

**or the sum of (a), (b), and (c)

f) THE OBSERVATIONS***

A	38.43	38.83	39.08	39.35
B	40.27	41.63	41.28	41.19
C	40.42	42.18	41.42	42.28
D	40.22	42.05	41.47	41.95
E	40.27	40.22	41.18	40.99

***or the sum of (a), (b), (c), and (d). Also the sum of (a) and (e).

g) THE CONDENSED ELEMENTARY ANALYSIS (for contrast)

	1	2	3	4	Bobbin
A	.27	-.51	-.06	-.02	-2.10
B	.03	.21	.06	-.26	-.02
C	-.32	.26	-.30	.33	.48
D	-.36	.29	-.09	.16	.32
E	.30	-.93	.23	-.19	-.29
Ends	-1.08	.10	-.10	.13	41.34

*In groups of 5.

of a full 5x4 table of 20 entries, and the structure of each piece of the partial description is shown by the pattern of repeated values. Once we are thoroughly familiar with the idea of elementary analysis, we have little need for such an expanded presentation. Indeed we then find its numerical repetitions wasteful. Before this stage of familiarity has been reached, however, the expanded presentation often helps to clarify our thinking.

5C. Visualizing the results

It is not enough to find an elementary analysis, we need to understand what we have found. We may be able to go fairly far toward understanding with numbers -- and should learn to do so -- but we ought to seek help from pictures.

Table 6 shows the results of an elementary analysis of a set of data new to us, reported by Scheffé in his 1959 book. The experiment studied the drained weight of cherries canned 24 hours earlier in terms of different weights -- tagged 0, 1, and 2 and called "fill" -- of raw cherries and different condition -- light, medium and dark, tagged L, M and D -- of that fruit. In the portion of the data given here the can was filled with syrup of 0° Brix, namely pure water.

With this small table, in which a variety of effects are large enough to be striking, we can do a fairly good job of interpretation from the table. We see that:

- drained weight increases with fill, more for the step from 0 to 1 than for the step from 1 to 2.

table 6 of chapter 5

Drained Weights of Canned Cherries

Fill of can	Color of fruit: L	M	D	
a) DATA				
0	55	95	169	
1	200	232	223	
2	233	285	291	
b) ELEMENTARY ANALYSIS				
0	-6	-5	45	-124
1	15	8	-25	0
2	-3	10	-8	51
	-39	0	24	224
c) FIT = PARTIAL DESCRIPTION				
0	61	100	124	
1	185	224	248	
2	236	275	299	

d) DISPLAY OF ELEMENTARY ANALYSIS WHEN ALONE

	L	M	D	Color
0	-6	-5	45	-124
1	15	8	-25	0
2	-3	10	-8	51
...				
Fill	-39	0	24	224

- drained weight increases with darkness, probably more from light to medium than from medium to dark.

- the residual for condition dark, fill 0 seems quite large in comparison with the others.

If we are to have a visual presentation of these results, we may well want to treat partial description and residuals separately.

When we come to portray the partial description, we need to note -- and use -- the central property of the partial description that comes with any elementary analysis:

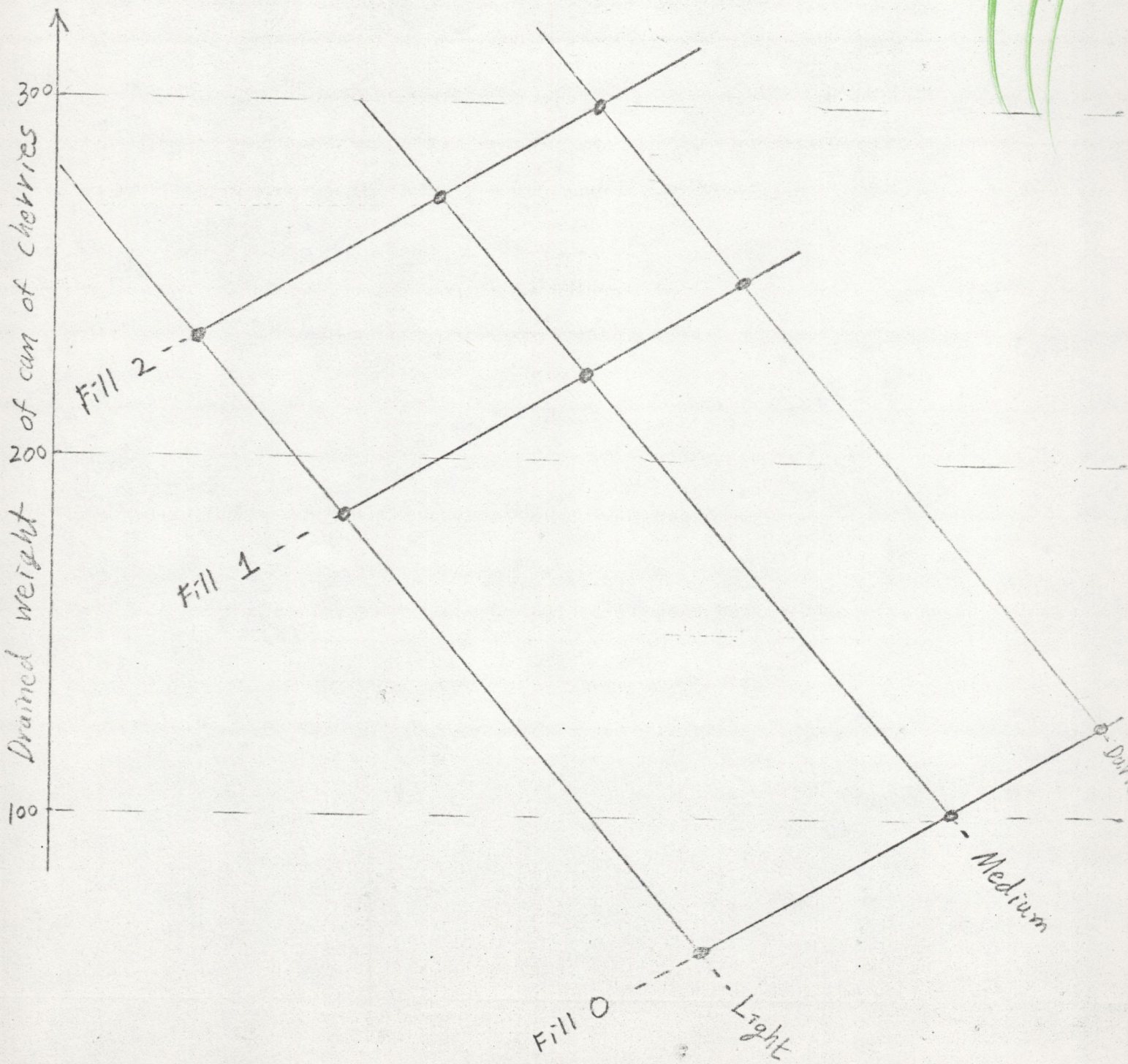
- the difference between each pair of two rows is the same in any column.

- the difference between each pair of two columns is the same in any row.

These are the properties of the heights of the crossing points formed when one family of parallel lines cuts a second family of parallel lines. Accordingly it is natural to try to represent such a fit in terms of two families of parallel lines. Plot 7 shows what can be done with the cherry data in this way. All the facts about the partial description are clear to see.

This is a type of plot most people are not used to. Horizontal position is only for maneuvering room. There is no horizontal scale. The vertical scale is meaningful, as is the fact that we have two families of parallel lines. But that is really all. Any other pair of families of parallel lines would do just as well, provided that their crossings came at exactly the same heights. We can draw any two lines, one from each family, locate the points on them from the vertical scale, and then draw the parallel lines. (See Section 5K for easy drawing

The fit = partial description for the drained weight of cherries.



In a case such as we have here, where row effects and column effects are large -- large by comparison with the residuals -- we can add the residuals to this picture without undue confusion. Plot 8 shows arrows leading straight up or straight down from the -- height of -- the fitted value to the -- height of -- the corresponding observation. We get from this picture a good idea of the relationship of the sizes of the fitted difference to the sizes of the residuals.

Unfortunately for the general usefulness of this sort of plot, it is quite clear that if either

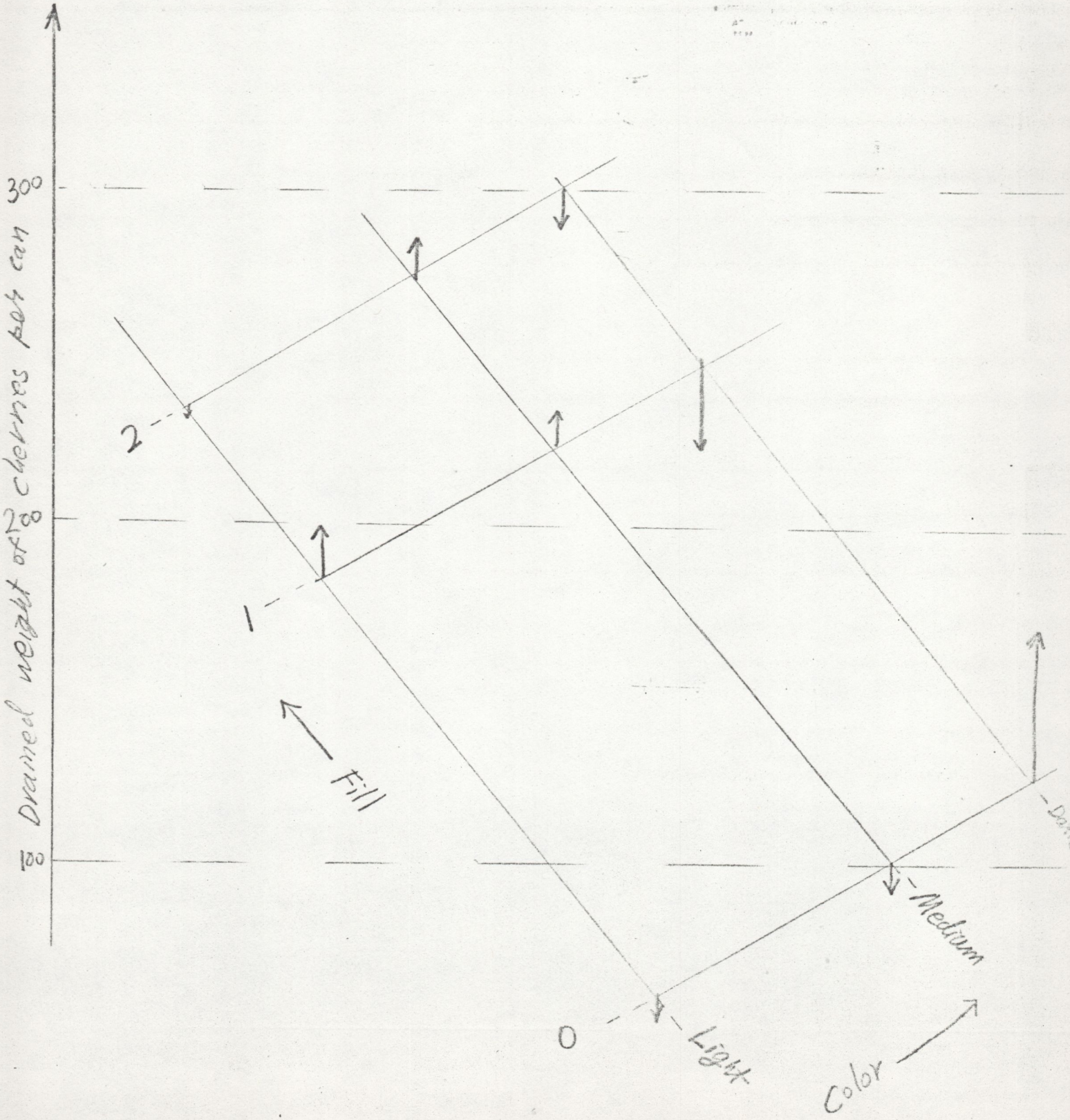
- we have many more data points, or
- the lines representing either set of stripes tend to bunch up too closely

this picture will become quite confusing. Plot 9, which shows the results of the elementary analysis of the thread-weight data of tables 3 to 5, shows the start of these effects. It is hard to learn about the pattern of residuals from this plot, though it can still be done. If we detach the residuals, and collect them in one corner of the picture, as in plot 10, we regain our clear view of the fitted partial description and preserve an idea of how sizes of residuals and fitted effects compare. For large or messy sets of data, this is about as complex a picture as we can afford.

So far, we do not have a good way to plot residuals in an appropriate pattern so that they can be appraised in direct context with the fitted partial description. The basic difficulty seems to be presenting a quite large amount of

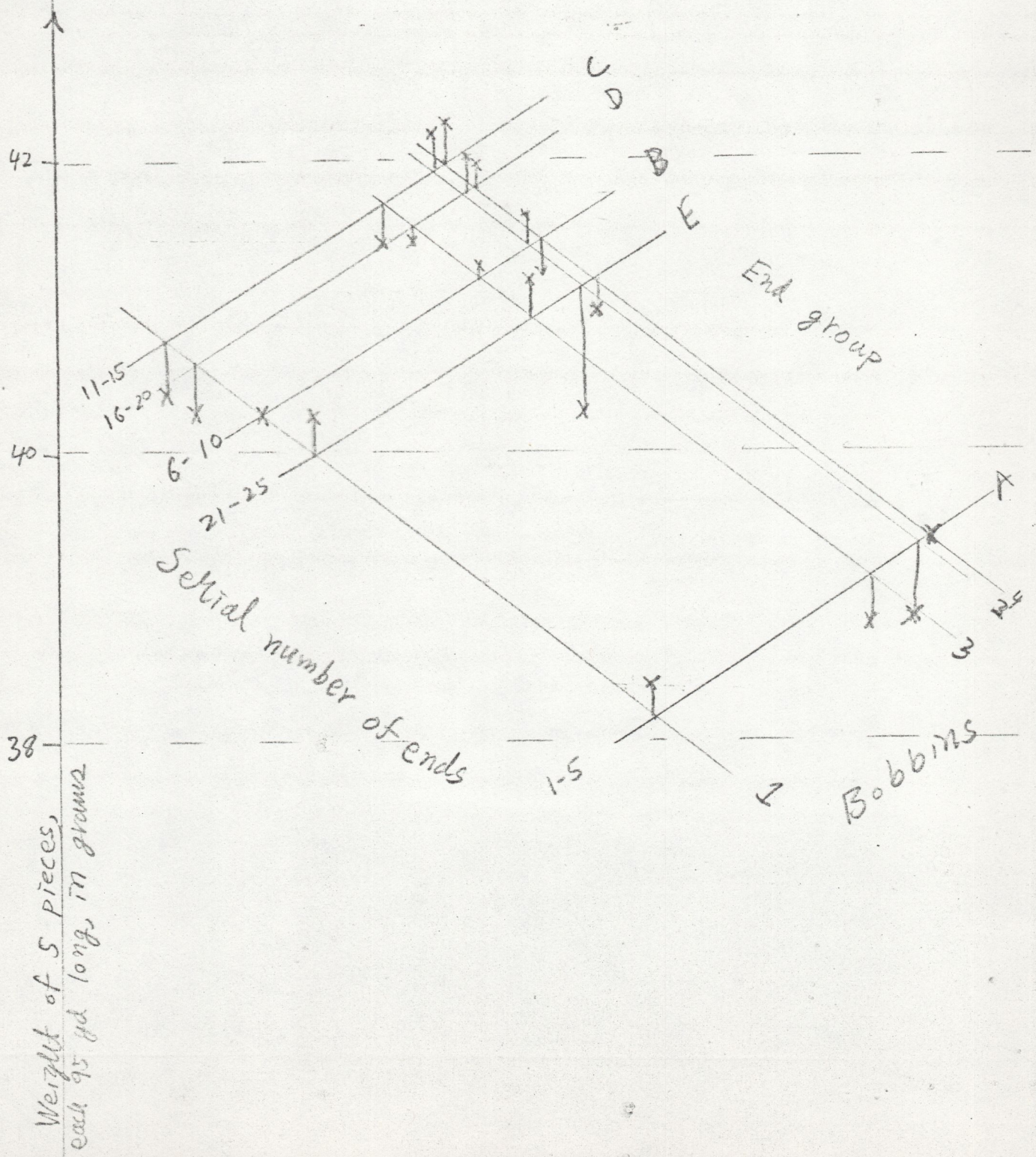
plot 8 of chapter 5

Fit and residuals for the drained weight of cherries.
(Observed weights at ends of arrows.)



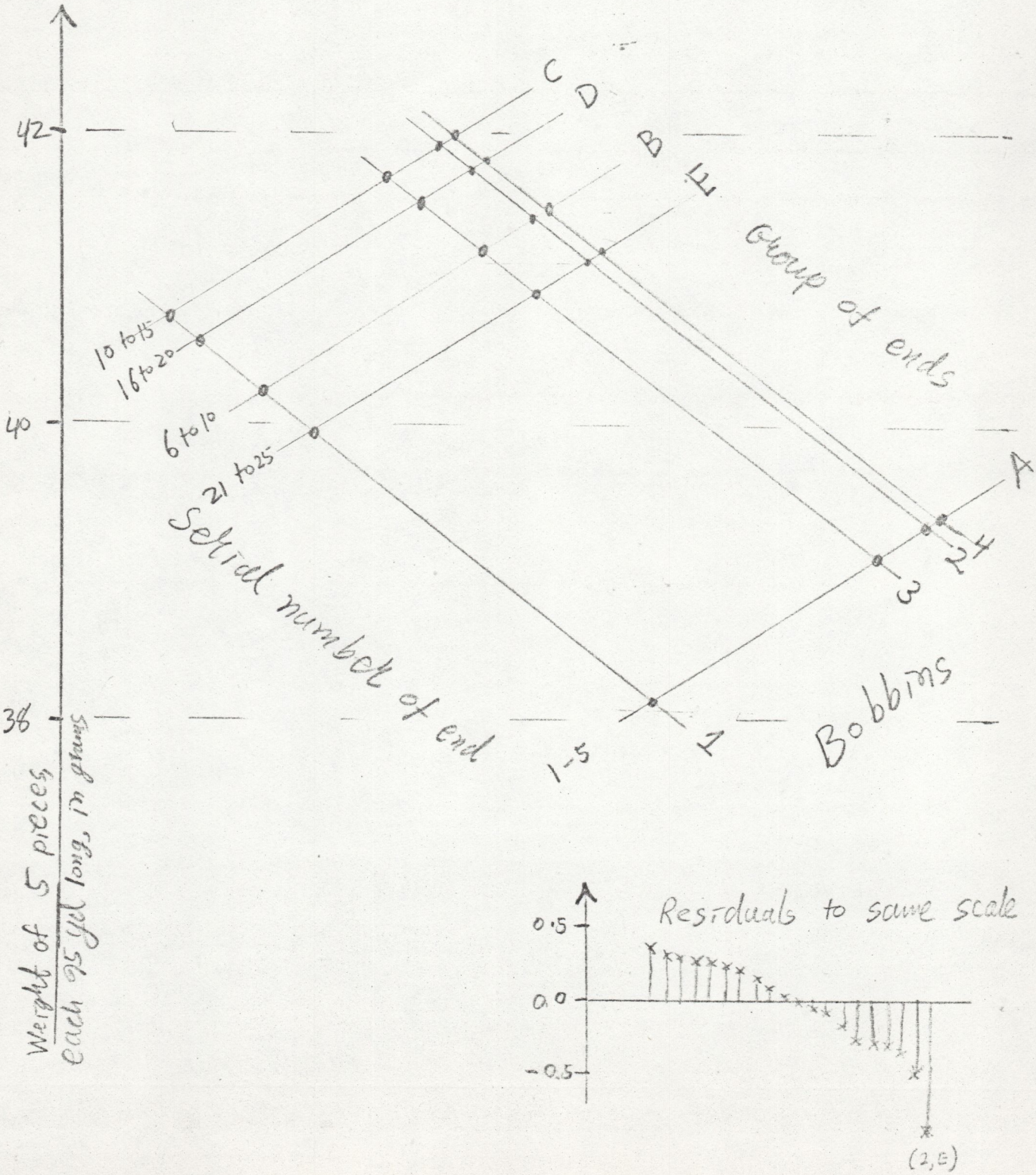
plot 9 of chapter 5

Combined display for the elementary analysis of tables 3 to 5



plot 10 of chapter 5

Fitted partial description of the elementary analyses of tables 3 to 5



information in a way that allows the viewer to turn his attention away from those parts of the situation that he wants not to see. Looking at parts of plot 10 -- or at aspects that are not confined to any one part, such as the comparative spacing of lines -- is not difficult. We have been able to deal effectively with partial descriptions, largely, it would seem, because they are much simpler in pattern than are residuals.

At least until someone has a new, good idea, we shall have to look hard at residuals in some other way.

5D. Coded arrays of residuals

Table 11 repeats portions of earlier displays of residuals for the thread-weight data, and carries evolution further. As we progress down table 11, the successive panels -- which carry almost the same information -- become clearer and clearer. If we want to be able to sweep our attention back and forth across a two-way table freely and easily we need to have simple entries. As this table shows, two decimals instead of one make an appreciable increase in complication. For some viewers, even the decimal points are a distraction.

We would like to be free to take full advantage of simplicity in displaying two-way tables of residuals. Just deciding to cut down to the first nonzero decimal will often not work well enough, as when all entries come out -1, -0, +0, or +1. We need to have a process that uses a small set of clearly marked, easily scanned tags. These tags need to be such that relative sizes are clearly distinguished, both

table 11 of chapter 5

Residuals of the Thread-Weight Data Yet Again

Group*	Bobbin 1	2	3	4
a) ADJUSTED VALUES, AS IN TABLE 3(a)				
A	41.61	40.83	41.28	41.32
B	41.37	41.55	41.40	41.08
C	41.02	41.60	41.04	41.67
D	40.98	41.63	41.25	41.50
E	41.64	40.41	41.57	41.15
b) RESIDUALS, AS IN TABLE 3(c)				
A	.27	-.51	-.06	-.02
B	.03	.21	.06	-.26
C	-.32	.26	-.30	.33
D	-.36	.29	-.09	.16
E	.30	-.93	.23	-.19
c) SAME, CUT BACK TO ONE DECIMAL				
A	.3	-.5	-.1	.0
B	.0	.2	.1	-.3
C	-.3	.3	-.3	.3
D	-.4	.3	-.1	.2
E	.3	-.9	.2	-.2
d) SAME, IN UNITS OF 0.1				
A	3	-5	-1	0
B	0	2	1	-3
C	-3	3	3	3
D	-4	3	-1	2
E	3	-9	2	-2

*of ends.

intuitively -- as the eye flies by -- and consciously. Clearly there can be many solutions, and much argument over relative merit.

We are about to present one scheme that seems to work fairly well. We urge its use until a better appears.

The symbols we shall use for tags are these:

```
HIGH
 HIG
 ·HI
 ··H
 ···
···
L··
LO·
LOW
LOWW
```

Intensity is carried by number of letters, while direction is told twice: once by which letters are used, and once to which side these letters are placed.

We want each of these tags to apply to an interval of values, with the locations and sizes of these intervals so fixed as to make the tags revealing. Again there could be many choices; again we vote for simplicity, basing our choice on the hinges of the batch of numbers to be displayed. The distance from one hinge to the other is too big to be a unit step, but half this distance is not. Accordingly, we work outward from the hinges -- and the center point midway between the hinges -- in steps of this size.

Table 12 shows how the coding rule looks for a very simple case where both hinges are very simple numbers.

table 12 of chapter 5

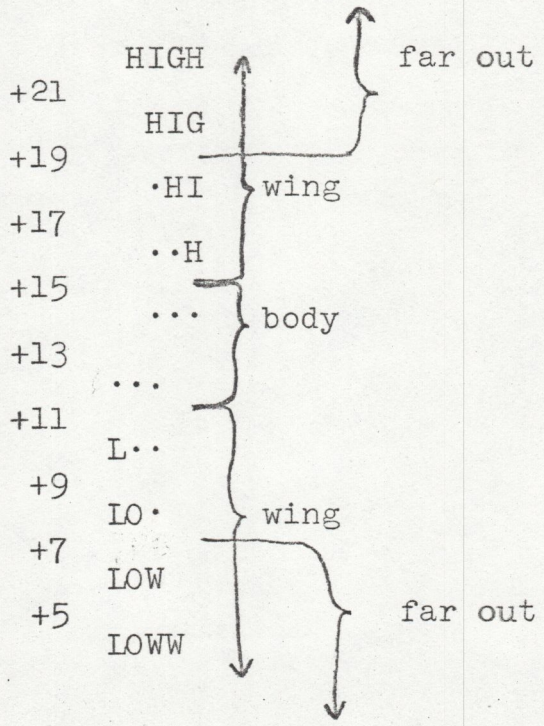
The Tags Used for Coded Values in a Simple Artificial Case

a) ASSUMED HINGES, CALCULATED UNIT STEP AND CUTTING POINTS

hinges (assumed):	+11,+15
midspread:	4 = 15 - 11
unit step:	2 = $\frac{1}{2}$(4)
central cutting points*:	+11,+13,+15
lower cutting points*:	+5,+7,+9
upper cutting points*:	+17,+19,+21

*all at unit step, here = 2.

b) CODING RULE AS A CRITICAL TABLE



Panel (b), in addition, stresses the relation of the tags to our previous cruder classification, namely:

HIG	or	HIGH	far out on the high side
··H	or	·HI	rest of the high wing
···	or	···	body
L··	or	LO·	rest of the low wing
LOW	or	LOWW	far out on the low side

Clearly, we have formed our tags by dividing each of our earlier classes in two parts, keeping as many lengths equal as we could. To do this, we work outward from the hinges in steps of this size, and add the point halfway between the hinges.

Table 13 shows the process and one result for a real example, the residuals just considered in table 11. The tendency of residuals to bow one way for Bobbin 1 but in the opposite direction for Bobbin 2 is now quite clear, as is the weakness of any similar effect for Bobbins 3 and 4.

Unfortunately, one table of coded residuals is rarely enough for a two-way table. Interchanging rows and columns has a considerable effect on what one can see. Table 14 shows this for the thread-weight residuals. We cannot see at all easily in the first panel what is so noticeable in the second. Since we never know which arrangement -- or which arrangements -- may be revealing for a specific set of data, we plan to always look at both.

There is a further possibility. We may want to look at the rows and columns in some other order. The thread-weight

table 13 of chapter 5

Coding the Thread-Weight Residuals

a) WING, HINGES, AND BODY ARRAY

.3	30
.2	716 93
.1	6
.0	36
-.0	629
-.1	9
-.2	6
-.3	206
-.4	
-.5	1
-.6	
-.7	
-.8	
-.9	3

b) STEM-AND-LEAF VERSION CALCULATION OF CODING CUTTING POINTS

hinges: -.28,.24

midspread: .52 = .24 - (-.28)

unit step: .26 = $\frac{1}{2}$(.52)

cutting points (spaced .26):

		HIGH
	+1.02	
		HIGH
	+.76	
		·HI
	+,50	
		··H
(hinge)	+.24	
		···
	-.02	
		···
(hinge)	-.28	
		L··
	-.54	
		LO·
	-.80	
		LOW
	-1.06	
		LOWW

c) CODED VERSION OF TWO-WAY TABLE OF RESIDUALS

End group	Bobbin 1	2	3	4
A	··H	L··	···	···
B	···	···	···	···
C	L··	··H	L··	··H
D	L··	··H	···	···
E	··H	LOW	···	···

table 14 of chapter 5

Coded Residuals With Rows and Columns Interchanged --
The Thread-Weight Example Again

a) ENDS AS COLUMNS

	A	B	C	D	E	Ends*
1	..H	...	L..	L..	..H	-1.08
2	L..	...	..H	..H	LOW	.10
3	...	...	L..	...	...	-.10
4	...	...	..H	...	...	.13
...						
Bobbin	-2.10	-.02	.48	.32	-.29	41.34

b) BOBBINS AS COLUMNS

	1	2	3	4	Bobbin
A	..H	L..	...	...	-2.10
B	...	...	...	...	-.02
C	L..	..H	L..	..H	.48
D	L..	..H	...	...	.32
E	..H	LOW	...	...	-.29
...					
Ends*	-1.08	.10	-.10	.13	41.34

*in groups of 5.

data was highly organized. Both rows and columns have a natural order. Where this is true we certainly want to look at the coded residuals arranged according to that order. Even so, this may not be enough. And when not enough, we would like to look at the coded residuals with rows and columns in as sensible an order as we know how to find. If no order came with the data -- or if we wish to set aside what came -- we have only the elementary analysis itself to use to fix orders.

The only simple way to do this is to use the orders of the row adjustments and column adjustments.

In the thread-weight data, the end-group adjustments were -- in tables 2(f) or 4(c) -- when we rearrange into order:

Adjustment:	-2.10	-.29	-.02	+.32	+.48
End group:	**A**	**E**	**B**	**D**	**C**

The bobbin adjustments, treated similarly, are

Adjustment:	-1.08	-.10	+.10	+.13
Bobbin:	**1**	**3**	**2**	**4**

Table **15** shows the two arrays of coded residuals that result.

table 15 of chapter 5

Tables of Coded Residuals in Stripe-Adjustment Order for the Thread-Weight Data

a) BOBBINS AS COLUMNS

41.34	-1.08	-.10	+.10	+.13	Ends
+.48	L··	L··	··H	··H	C
+.32	L··	···	··H	···	D
-.02	···	···	···	···	B
-.29	··H	···	LOW	···	E
-2.10	··H	···	L··	···	A
Bobbin	1	3	2	4	

b) END GROUPS AS COLUMNS

41.34	-2.10	-.29	-.02	+.32	+.48	Bobbin
.13	···	···	···	···	··H	4
.10	L··	LOW	···	··H	··H	2
-.10	···	···	···	···	L··	3
-1.08	··H	··H	···	L··	L··	1
Ends	A	E	B	D	C	

In this instance, rearranging rows and columns has made simple and striking an appearance that we had noted without much understanding in one of the two arrays examined earlier. In either of the arrays of table 15, we find the H's at the NE and SW corners, the L's at the NW and SE corners. While we do not yet understand how to interpret the message, there is little doubt that it is clear and vigorous.

Thus we resolve to regard two pairs of coded residual displays -- one like table 14 and one like table 15 -- as the normal procedure when we want to look hard at a two-way table of residuals. If the data comes with no order at all, we can omit the first pair. Otherwise, we plan to use both pairs.

5E. <u>Yes, it can be done with means, but -</u>

Hand calculation of midmean differences, square combining tables, and the steps of fuller treatment can become tedious, especially when there are many rows or many columns. Two alternatives lie before us when we face this:

- We can turn the computation over to a programmed computer -- if we have the computer and the program -- with ease and convenience.

- We can try to find some simpler arithmetic which will, in at least some cases, come to an elementary analysis different in detail but similar in general behavior.

The situation is quite similar to that of a man who:

- does not know the stem-and-leaf technique, and thus finds putting lists of numbers in order tedious or painful.

- has a number of batches to summarize, each of 50 numbers.

- regrets having to put them in order as a necessary part of calculating either the midmean or the median.

- can add up numbers easily.

His choices are also two -- to put the calculations on a computer or to try to get away with the arithmetic mean. If there are few if any straggling values in his batches, the arithmetic mean may work well for him. If there tend to be too many far-out values, he cannot afford its use.

So there, also here. We can try arithmetic means on two-way tables. Sometimes -- when the tables are well-behaved -- we can do quite well with them. At another extreme, analysis by arithmetic means will leave the most important appearances concealed. Elementary analysis by means can work -- but need not.

Table 16 sets out the computations, using still the thread-weight data. The calculations clearly give us an elementary analysis, and take few arithmetic steps, if checks are not needed. They do not, however, give us much "feel" of what is going on as we calculate. Partly as a consequence, we seem to need checks.

If we are prepared to give up symmetry and move step by step, we can reach the same end in a clearer way, as table 17 shows. Except for differences in rounding, the tables of residuals -- in the new table called "second residuals" -- and of fits are the same. If we are to do

table 16 of chapter 5

Elementary Analysis of the Thread Weights
By Arithmetic Means

End Group	Bobbin 1	2	3	4	Sum	Mean
a) THE DATA, SUMS, AND MEANS						
A	38.43	38.83	39.08	39.35	155.69	38.92
B	40.27	41.63	41.28	41.19	164.37	41.09
C	40.42	42.18	41.42	42.28	166.30	41.58
D	40.22	42.05	41.47	41.95	165.69	41.42
E	40.27	40.22	41.18	40.99	162.66	40.66
Sum	199.61	204.91	204.43	205.76	814.71	(203.67)
Mean	39.92	40.98	40.89	41.15	(162.94)	40.74
b) STRIPE ADJUSTMENTS, each = STRIPE MEAN minus GRAND MEAN						
A					-1.82	
B					.35	
C					.84	
D					.68	
E					-.08	
Adj	-.82	.24	.15	.41	(40.74)	
c) FIT equals PARTIAL DESCRIPTION						
A	38.10	39.16	39.07	39.33	(155.66)	(38.92)
B	40.27	41.33	41.24	41.50	(164.34)	(41.09)
C	40.76	41.82	41.73	41.99	(166.30)	(41.58)
D	40.60	41.66	41.57	41.83	(165.66)	(41.42)
E	39.84	40.90	40.81	41.07	(162.62)	(40.66)
Sum	(199.57)	(204.87)	(204.42)	(205.72)		
Mean	(39.91)	(40.97)	(40.88)	(41.14)		
d) RESIDUALS						
A	.33	-.33	.01	.02	(.03)	
B	+.00	.30	.04	-.31	(.03)	
C	-.34	.36	-.31	.29	(.00)	
D	-.38	.39	-.10	.12	(.03)	
E	.43	-.68	.37	-.08	(.04)	
	(+.04)	(.04)	(.01)	(.04)		

table 16 (Cont'd) - 2

e) CHECKING THE CALCULATION

814.71 is the sum of either the column of row sums or of the row of column sums

$$40.74 = \frac{814.71}{(4)(5)} = \frac{203.67}{5} = \frac{162.94}{4} \quad \text{(near enough)}$$

$-1.82 + .35 + .84 + .68 - .08 = -.03$, near enough to .00

$-.82 + .24 + .15 + .41 = -.02$, near enough to .00

Marginal sums and means in (c) check those in (a), marginal sums in (d) are .00, nearly enough (and can be checked by differences of marginal sums in (a) and (b)).

table 17 of chapter 5

Step-By-Step Elementary Analysis of the Thread Weight Data by Means

End group	Bobbin 1	2	3	4
a) THE DATA, COLUMN SUMS and COLUMN MEANS				
A	38.43	38.83	39.08	39.35
B	40.27	41.63	41.28	41.19
C	40.42	42.18	41.42	42.28
D	40.22	42.05	41.47	41.95
E	40.27	40.22	41.18	40.99
Sum	199.61	204.91	204.43	205.76
Mean	39.92	40.98	40.89	41.15

b) FIRST RESIDUALS = DATA minus ITS COLUMN MEAN

					Sum	Means
A	-1.49	-2.15	-1.81	-1.80	-7.25	-1.81
B	.35	.65	.39	.04	1.43	.36
C	.50	1.20	.53	1.13	3.36	.84
D	.30	1.07	.58	.80	2.75	.69
E	.35	-.76	.29	-.16	-.28	-.07
(Sum)	(.01)	(.01)	(-.02)	(.01)	(.01)	(.01)

c) SECOND RESIDUALS = FIRST RESIDUAL minus ITS ROW MEAN

A	.32	-.34	.00	.01	(-.01)
B	-.01	.29	.03	-.32	(-.01)
C	-.34	.36	-.31	.29	(.00)
D	-.39	.38	-.11	.11	(-.01)
E	.42	-.69	.36	-.09	(.00)
	(.00)	(.00)	(-.03)	(.00)	(-.03)

d) FIT = COLUMN MEANS of (a) plus ROW MEANS of (b)

A	38.11	39.17	39.08	39.34
B	40.28	41.34	41.25	41.51
C	40.76	41.82	41.73	41.99
D	40.61	41.67	41.58	41.84
E	39.85	40.91	40.82	41.08

table 17 (Cont'd) - 2

e) COLUMN ADJUSTMENTS = COLUMN MEANS minus THEIR MEAN

*	-.82	.24	.15	.41

Here THEIR MEAN = $\frac{1}{4}(39.92+40.98+40.89+41.15) = 40.74$.

f) ELEMENTARY ANALYSIS

	1	2	3	4	⋮	Bobbin
A	.32	-.34	.00	.01		-1.81
B	-.01	.29	.03	-.32		.36
C	-.34	.36	-.31	.29		.84
D	-.39	.38	-.11	.11		.69
E	.42	-.69	.36	-.09		-.07
...						
Ends	-.82	.24	.15	.41		40.74

g) EXAMPLE OF DETAILED E-A (ELEMENTARY ANALYSIS)

$$38.43 = (40.74) + (-1.81) + (-.82) + (.32)$$

elementary analysis in this way -- with means and by hand -- we are almost always better off following the pattern of table 7, or proceeding similarly but taking rows out before columns.

There are two reasons why elementary analysis by means may not serve us well:

- there may be one or more way-out values in the data.
- there may be missing values.

The problem with means, so far as way-out values are concerned, is exactly similar here as it was for batches. Means are too sensitive to single extreme values. Consider, for example, the effect of increasing the thread weight for bobbin 4, group A by 20 units. (So large a change here could hardly come from any cause but human error.) Table 18 shows the original and modified data and two sets of elementary analyses, one by midmean differences, the other by means. Since our main concern is with the changes -- most particularly with the changes in residuals, table 19 shows these changes.

Whichever of these tables we look at, we see utter disintegration of the elementary analysis using means alone as a result of this single very extreme value, but only a quite small influence on the elementary analysis using midmeans.

Whenever single very extreme values are to be feared, the use of means alone in making elementary analyses is also to be feared.

table 18 of chapter 5

Effects on Elementary Analyses of Large Change in One Data Value

End group	ORIGINAL DATA Bobbin 1	2	3	4		MODIFIED DATA Bobbin 1	2	3	4	
a) DATA										
A	38.43	38.83	39.08	39.35		38.43	38.83	39.08	59.35	
B	40.27	41.63	41.28	41.19		40.27	41.63	41.28	41.19	
C	40.42	42.18	41.42	42.28		40.42	42.18	41.42	42.28	
D	40.22	42.05	41.47	41.95		40.22	42.05	41.47	41.95	
E	40.27	40.22	41.18	40.99		40.27	40.22	41.18	40.99	
b) ELEMENTARY ANALYSIS BY MIDMEAN DIFFERENCES WITH MIDMEAN POLISH										
A	.27	-.51	-.06	-.02	-2.10	.26	-.56	-.09	19.70	-2.09
B	.03	.21	.06	-.26	-.02	.06	.20	.07	-.50	-.01
C	-.32	.26	-.30	.33	.48	-.25	.29	-.25	.13	.60
D	-.36	.29	-.09	.16	.32	-.34	.27	-.09	-.09	.41
E	.30	-.93	.23	-.19	-.29	.35	-.92	.26	-.41	-.41
*	-1.08	.10	-.10	.13	41.34	-1.11	.11	-.11	.37	41.34
c) ELEMENTARY ANALYSIS BY MEANS										
A	.32	-.34	.00	.01	-1.81	-3.68	-4.34	-4.00	12.01	2.19
B	-.01	.29	.03	-.32	.36	.99	1.29	1.03	-3.32	-.64
C	-.34	.36	-.31	.29	.84	.66	1.36	.69	-2.71	-.16
D	-.39	.38	-.11	.11	.69	.61	1.38	.89	-2.89	-.31
E	.42	-.69	.36	-.09	-.07	1.42	.31	1.36	-3.09	-1.07
*	-0.82	.24	.15	.41	40.74	-1.82	-.76	-.85	3.41	41.74

table 19 of chapter 5

Effects on Residual and on Fits of Large Change in One Data Value

	Bobbin			
	1	2	3	4
a) ASSUMED CHANGE IN DATA				
A	0	0	0	+20.00
B	0	0	0	0
C	0	0	0	0
D	0	0	0	0
E	0	0	0	0
b) EFFECT ON RESIDUALS BY MIDMEAN DIFFERENCES				
A	-.01	-.05	-.03	19.72
B	+.03	-.01	+.01	-.24
C	+.07	+.03	+.05	-.20
D	+.02	-.02	.00	-.25
E	+.05	+.01	+.03	-.22
c) EFFECT ON RESIDUALS BY MEANS				
A	-4.00	-4.00	-4.00	12.00
B	1.00	1.00	1.00	-3.00
C	1.00	1.00	1.00	-3.00
D	1.00	1.00	1.00	-3.00
E	1.00	1.00	1.00	-3.00

The situation about missing values is even more serious. Table 20 deals with an oversimple example. Whatever value we think should play the role of the grand mean, the column mean shows column 5 different from the other columns by 80 units, while the data table is clear that there should be no indicated difference.

Surely the analysis by means can be fixed up to avoid this difficulty. As we shall see in section 5G, this is so, but the fixing-up changes the nature of the procedure so much that we will need to rethink our position.

5F. Immunity and transparency

It is time to ask what our methods of analysis are really being asked to do, and how they manage to do what they are asked. When we seek column adjustments, separately or together with a general typical, what is it that we require? We have seen various answers: by midmean differences with and without midmean polish (= fuller treatment), by means alone. Clearly the general concept has more than one example. Probably it can be defined by properties, but which ones?

The simplest property is one we might overlook. It reads:

> - if the data CAN be fitted exactly by a rows-columns-and-general partial description, our fitting procedure will fit it exactly.

Once stated, it is hard to avoid admitting that this is a basic property.

table 20 of chapter 5

Failure of the Mean Analysis With a Missing Value

	1	2	3	4	5	Sum	Mean
a) THE DATA, BORDERED WITH SUMS AND MEANS							
A	400	400	400	410	x	1610	??
B	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0
Sum	400	400	400	410	0	1610	
Mean	80	80	80	82	0		?
b) THE COLUMN ADJUSTMENTS							
(*)	16	16	16	18	-64		
(**)	13	13	13	15	-67		

(*) if grand mean taken as $\frac{1}{5}$ (80+80+80+82+0) = 1602/25.

(**) if grand mean taken as 1602/24.

We ought next to look at changes. If we start with data that can be exactly fitted and go to data that can be exactly fitted, the difference can be exactly fitted too -- at least for the kinds of partial description we are considering in this chapter. It follows from our condition that the change in fit exactly matches the change in data. We could require this for a change starting with any data, not just with data that can be exactly fitted. After we think over, as well as we can, the implications of doing this, we decide to insist on this more general condition.

There are various alternate ways to state the resulting condition, ways that are logically equivalent though different in appearance. These include:

- if the data is changed in a way that is exactly fittable, the change in the fit is identical -- except for the effects of rounding of numbers -- with the change in the data.

- if the data is changed in a way that is exactly fittable, the new residuals are identically the same as the old residuals -- except for rounding.

- some two-way tables are possible tables of residuals, most are not -- by subtracting possible partial descriptions from any data table we can reach only one table that is a possible table of residuals (again except for effects of rounding numbers).

- if we take the residuals as new data, the new partial description is zero and the new residuals are the same as the old (again except for effects of rounding).

Each reader will find his insight into what is going on greatly improved by the process of convincing himself that each of these conditions is logically equivalent to the others.

One easy consequence of these properties is worth note. Suppose that we have made an elementary analysis by any procedure that has these properties, and that we want to change a few values and reanalyze. All that we need do is to:

- take the old residuals,
- make the changes in them matching the desired changes in data,
- reanalyze the modified residuals.

If we do this, then:

- the new residuals are the residuals for the analysis of the modified data.
- the partial description for the modified data is the SUM of (i) the original partial description and (ii) the partial description found for the modified residuals.

This fact often saves arithmetic.

We have written down these conditions in a quite general way. The only mention of anything specific to a two-way table is in describing the tables among which some are possible residuals. Here, if we were concerned with some other problem, we could make the appropriate change of words. Once this is done, all four forms of the condition are at our service, all four are still equivalent.

One name we could apply to this condition is "transparency" . Any change in the data of a specified sort -- here anything fittable by the partial description -- passes through unchanged into the partial description. None of it is absorbed into the residual.

The possibility of having transparency is intimately bound up with the fact that if we take two possible partial descriptions and add them up, observation by observation, the result is another partial description. Not all sets of possible partial descriptions that we may need to use have this property, but many, many do.

In general language, then, so long as we have this property of possible partial descriptions, it seems natural to require that:

- if all the data are zero, so too is the partial description.
- the process of partial description is transparent to changes that are themselves possible partial descriptions.

These conditions are equivalent to the others we have been discussing so long as:

- one partial description plus another partial description is itself a partial description.

Our concern here is with two-way tables. How can we reformulate these conditions so that they apply more specifically and clearly to this case? For the stripe adjustments this is rather easy. We have only to say how these respond to changes that are constant by stripe, since any possible partial description can be written -- in various ways -- as a sum of something constant by rows and something constant by columns. In these terms we have to require two things: sometimes immunity, sometimes transparency.

In general, we require:

- each set of stripe adjustments is unaffected by -- is immune to -- any change that is constant by stripes of the other kind.

- each set of stripe adjustments is transparent, except for a possible constant difference absorbed in the general typical, to any change that is constant by stripes of the same kind.

In order to gain from concreteness, let us write these two out in each of the two detailed forms we can give them for a two-way table:

- row adjustments are immune to any change that is constant by columns.

- column adjustments are immune to any change that is constant by rows.

- any change constant by rows changes the row adjustments by amounts that either are the same as the corresponding changes or differ from those changes by a common constant.

- any change constant by columns changes the column adjustment by amounts that either are the same as the corresponding changes or differ from these changes by a common constant.

The art of finding good stripe adjustments, then, begins by saying to what they shall be wholly unresponsive and to what they shall be wholly responsive. Of the two kinds of conditions, the first is often -- perhaps surprisingly -- the more important. Transparency is important, but immunity can be even more important.

5G. The inner workings

How then do the various methods we have used to find elementary analyses -- whether or not called such -- work in detail? How do they gain immunity? How transparency?

For analysis by midmean differences, the answers are easy. The central step is the first one -- forming a

difference, say a difference of values in two specified columns but in the same row. Here both immunity and transparency are gained at one step. A major purpose of the rest of the computation can then be to preserve them.

Forming this difference gives immunity, for any change that is constant by rows cancels out identically. Forming this difference leads toward transparency, since any change constant by columns changes this difference by the correct amount, the difference in changes for the columns specified.

Once we have these differences, two steps are in order:

- combining differences of the same sort.
- resolving any incompatibilities among the summarized differences of all the various sorts.

In elementary analysis by midmean differences we do the first by forming midmeans, the second through either the square combining table alone or through this table followed by fuller treatment through midmean polishing. When we check up, we find that immunity is preserved -- which is quite obvious -- and that transparency is attained.

Clearly there is no restriction to midmeans in such a process. We can use medians or means -- to take but two examples -- to summarize differences of the same sort. The square combining table will accept the results with just as good grace as it does for midmeans. Elementary analysis by median differences and elementary analysis by mean differences are perfectly reasonable processes. We adopted midmean differences as a matter of choice, not necessity. We adopted it

for the same reason -- good resistance to way-out values combined with reasonable effectiveness in taking account of most values -- that we adopted midmeans as the summary for batches.

It turns out, for simple reasons, that elementary analysis by mean differences produces certain simplifications. Table 2I repeats table 1 using mean differences. Interestingly enough, the combining residuals all vanish. This says something very simple: the original mean differences were compatible. If this is so at all frequently:

- we can certainly take advantage of this, whenever it is appropriate to use mean differences and it is so.
- we ought to understand why and when it is so, since we are likely to be able to take further advantage of the understanding.

To take trivial advantage of the compatibility, we notice that it is enough to look at only three sets of differences, say 1-2, 2-3, and 3-4, for from their means we can calculate the means of the others. If these three are -1.06, .10 and -.27, as in this example, and if the column adjustment for bobbin 1 is g, then those for bobbins 2, 3, and 4 are:

$$(g) - (-1.06) = g + 1.06$$
$$(g+1.06) - (.10) = g + 0.96$$
$$(g+0.96) - (-.27) = g + 1.23$$

and the sum of the four is thus

$$4g + 3.25$$

table 2I of chapter 5

A Partial Analysis by Mean Differences of
Daniels's Weights of Thread

a) THE DATA (see table 1)

End groups	Bobbin 1	2	3	4
A	38.43	38.83	39.08	39.35
B	40.27	41.63	41.28	41.19
C	40.42	42.18	41.42	42.28
D	40.22	42.05	41.47	41.95
E	40.27	40.22	41.18	40.99

b) DIRECT COMPARISONS AMONG BOBBINS

	1-2	1-3	1-4	2-3	2-4	3-4
	-.40	-.65	-.92	-.25	-.52	-.27
	-1.36	-1.01	-.92	.35	.44	.09
	-1.76	-1.00	-1.86	.76	-.10	-.86
	-1.83	-1.25	-1.73	.58	.10	-.48
	0.05	-.91	-.72	-.96	-.77	.19
sum	-5.30	-4.82	-6.15	.48	-.85	-1.33
mean	-1.06	-.96	-1.23	.10	-.17	-.27

c) SQUARE COMBINING TABLE

Minus bobbin	Plus Bobbin 1	2	3	4
1	0	1.06	.96	1.23
2	-1.06	0	-.10	.17
3	-.96	.10	0	.27
4	-1.23	-.17	-.27	0
Sums	-3.25	.99	.59	1.67
Means	-.81	.25	.15	.42

d) FITTED DIFFERENCES

1	0	1.06	.96	1.23
2	-1.06	0	-.10	.17
3	-.96	.10	0	.27
4	-1.23	-.17	-.27	0

Example: -1.06 = (-.81) - (.25)

table 21 (Cont'd) - 2

e) COMBINING RESIDUALS

	1	2	3	4
1	0	.00	.00	.00
2	.00	0	.00	.00
3	.00	.00	0	.00
4	.00	.00	.00	0

Since this sum is forced to zero by the square combining table, we must have

$$g = -\frac{3.25}{4} = -.81$$

and the four column adjustments are

$$-.81, \quad .25 \quad .15 \quad .42$$

as in table 21. Clearly we have saved a considerable amount of arithmetic. Why could we? Why is there compatibility?

Let us look at the mean difference for 1-2 in more detail. It can be written in many equivalent forms. Some of them are:

$$\frac{1}{5}[(-.40) + (-1.36) + (-1.76) + (-1.83) + (0.05)]$$

$$= \frac{1}{5}[(38.43-38.83) + (40.27-41.63) + (40.42-42.18 + (40.22-42.05) + (40.27-40.22)]$$

$$= \frac{1}{5}[(38.43+40.27+40.42+40.22+40.27) - (38.83+41.63+42.18 +42.05+40.22)]$$

$$= \frac{38.43+40.27+40.42+40.22+40.27}{5} - \frac{38.83+41.63+42.18+42.05+40.22}{5}$$

When we compare the first with the last, and put everything in words, we see that, at least in this example:

|| mean difference ≡ difference in means

Since this sum is forced to zero by the square combining table, we must have

$$g = -\frac{3.25}{4} = -.81$$

and the four column adjustments are

$$-.81, \quad .25 \quad .15 \quad .42$$

as in table 21. Clearly we have saved a considerable amount of arithmetic. Why could we? Why is there compatibility?

Let us look at the mean difference for 1-2 in more detail. It can be written in many equivalent forms. Some of them are:

$$\frac{1}{5}[(-.40) + (-1.36) + (-1.76) + (-1.83) + (0.05)]$$

$$= \frac{1}{5}[(38.43-38.83) + (40.27-41.63) + (40.42-42.18 + (40.22-42.05) + (40.27-40.22)]$$

$$= \frac{1}{5}[(38.43+40.27+40.42+40.22+40.27) - (38.83+41.63+42.18 +42.05+40.22)]$$

$$= \frac{38.43+40.27+40.42+40.22+40.27}{5} - \frac{38.83+41.63+42.18+42.05+40.22}{5}$$

When we compare the first with the last, and put everything in words, we see that, at least in this example:

|| mean difference ≡ difference in means

A little thought shows that this has to be always so. Accordingly, we did not even need to calculate differences. We could have reached exactly the same column adjustments by taking the means of the various columns and then subtracting a suitable constant -- as we did in panels (a) and (b) of table 16.

We can do this for a simple reason -- exactly the same set of rows enter into the differences for each pair of columns. When this is not the case, when we have incomplete data, there is no match of difference in means to mean differences, and hence no short cut. Indeed it is easy to see that there cannot be a match of this kind, because a simple example shows that the mean differences need not be compatible.

Table 22 illustrates this, using a slight modification of the data of table 20.

Thus we at least understand:

- why our short-cut analyses using means work for complete tables -- because it is there equivalent to the use of mean differences.

- how to "fix it up" for incomplete tables -- go over to mean differences.

Let us now rethink the position: Since elementary analysis by mean differences requires all the individual steps of analysis that elementary analysis by midmean differences needs, we gain no short-cut advantages once we have incomplete data. As arithmetic short-cutting was the only really favorable property for elementary analysis by means, we may as well give it up as an approach to incomplete data.

table 22 of chapter 5

Partial Analysis, by Mean Differences, of Hypothetical Incomplete Data
(Compare table 20)

a) THE DATA

	1	2	3	4	5
A	400	400	400	410	xxx
B	0	0	0	0	0
C	0	0	0	0	0
D	0	0	0	0	0
E	0	0	0	0	0

b) DIRECT COMPARISON OF COLUMNS

	*	**	***
	0	-10	XXX
	0	0	0
	0	0	0
	0	0	0
	0	0	0
Sum	0	-10	0
Mean	0	-2	0

* among any two of 1, 2 and 3
** between 1, 2, or 3 and 4
*** between 1, 2, 3, or 4 and 5

c) SQUARE COMBINING TABLE

Minus column	Plus column 1	2	3	4	5
1	0	0	0	2	0
2	0	0	0	2	0
3	0	0	0	2	0
4	-2	-2	-2	0	0
5	0	0	0	0	0
Sum	-2	-2	-2	+6	0
Mean	-0.4	-0.4	-0.4	+1.2	0.0

d) FITTED DIFFERENCES

1	0	0	0	1.6	0.4
2	0	0	0	1.6	0.4
3	0	0	0	1.6	0.4
4	-1.6	-1.6	-1.6	0	-1.2
5	-0.4	-0.4	-0.4	1.2	0

table 22 (Cont'd) - 2

	1	2	3	4	5
e) COMBINING RESIDUALS					
1	0	0	0	0.4	-0.4
2	0	0	0	0.4	-0.4
3	0	0	0	0.4	-0.4
4	-0.4	-0.4	-0.4	0	1.2
5	0.4	0.4	0.4	-1.2	0
f) THE COLUMN ADJUSTMENTS					
	-0.4	-0.4	-0.4	1.2	0.0

5H. Compromises

We may not trust elementary analysis by means because we fear the effects of way-out values. (We may not be able to use it because the data is incomplete, from this there may be little escape.) Is there not, however, some way to do less arithmetic than the full analysis by midmean differences? Sometimes the answer is yes.

Let us go back to the property of transparency. Suppose that we have a method of finding column adjustments that has this property. Suppose we find -- and subtract from the data -- these column adjustments. What then? At least two points are important:

- the change due to this subtraction is constant by columns, and should not be allowed to change any row adjustments -- in particular we can calculate row adjustments appropriate to the original values from these new column-adjusted values.

- since fitting column adjustments is transparent, any change in the original data that is constant by columns will leave the column-adjusted values unaffected -- thus in seeking to find row adjustments from the column-adjusted values we can forget about immunity, since this procedure makes it automatic.

Perhaps we can save some arithmetic by doing column adjustment carefully -- probably by midmean differences -- and then using the column adjusted values as the basis for a simpler calculation of row adjustments. Actually, we have already done this in one instance, before we met the words "elementary analysis".

If we turn back to section 4E, where we were working with simple comparisons made within blocks, we made an elementary analysis without saying so. We can perfectly well treat blocks as something among which we are interested in comparisons. Our interests are usually much weaker than they are for the comparisons for which the data was taken, but they may well be there. In table 15 of chapter 4, we used midmean differences (see table 7 of chapter 4) to find adjustments for columns (there fertilizer treatments), and applied them to find adjusted values. We then made one of the simplest resistant analyses of these adjusted values we know how to do, we took midmeans of each row.

Table 23 emphasizes the situation by giving data and two forms of elementary analysis, differing in what summaries of column adjustments and of row adjustments have been forced to vanish, one forced to mean zero, the other forced to midmean zero.

This type of calculation saves arithmetic when we deal with tall, thin tables. With c columns, the use of differences means that each observation appears in c-1 differences, and thus each observation has to be handled c-1 times. With r rows, and the use of differences, each observation appears in r-1 differences and each observation has to be handled r-1 times. (We also have to do combining, but this is only a small part of the calculation when r and c are both large.)

If we form column-adjusted values and then midmeans by row, each observation appears in one adjusted value and thence in one midmean. Each observation thus has to be

table 23 of chapter 5

Elementary Analyses Corresponding to Tables 15 and 7 of Chapter 4; The Sugar-Beet Data Again

Block	0, none	1, phosphate	2, phosphate and nitrate	
a) OBSERVED VALUES				
1	2.45	6.71	6.48	
2	2.25	5.44	7.11	
3	4.38	4.92	5.88	
4	4.35	5.23	7.54	
5	3.42	6.74	6.61	
6	3.27	4.74	8.86	
b) ELEMENTARY ANALYSIS FROM TABLE 15 of CHAPTER 4				
1	-.51	1.35	-.19	-.25
2	-.75	.08	.44	-.25
3	1.57	-.25	-.60	-.44
4	.57	-.91	.09	.53
5	-.08	.88	-.56	.25
6	-.17	-1.06	1.75	.19
	-2.01	0.35	1.66	5.20
c) ELEMENTARY ANALYSIS WITH ADJUSTED ADJUSTMENTS				
1	-.51	1.35	-.19	-.23
2	-.75	.08	.44	-.23
3	1.57	-.25	-.60	-.42
4	.57	-.91	.09	.55
5	-.08	.88	-.56	.27
6	-.17	-1.06	1.75	.21
	-2.36	.00	1.31	5.59

handled twice. The two approaches call for about the same amount of handling when r-1 = 2. Once r is appreciably larger than 3, then gain in handling becomes substantial.

For our example we have to handle individual numbers as follows:

all by differences: 2 = c-1 times for columns
5 = r-1 times for rows
(2 combining tables, etc.)

columns by differences, but rows by midmeans of adjusted values:

2 = c-1 times for columns
2 times for rows
(1 combining table, etc.)

In this example, the compromise has saved more than half the arithmetic. In a 3x30 or 4x40 table, the saving is by more than 85%. Thus, especially when working by hand, the compromise can have very great advantages.

When we face larger tables, and must analyze them by hand, it can help to have looked deeper into the possibilities.

Indeed, if we have:

- a complete table, and
- a fear of way-out values,

we can use:

- analysis by means followed by midmean polish.

We can, indeed, combine transparency, resistance, and reasonably compressed arithmetic.

Why? The first step, analysis by means, provides transparency without resistance. Any change constant by stripe flows wholly into the fit. Later steps, so long as they look only at the residuals from the first step, removing part of these first residuals and adding what has been removed to the fit, cannot affect this transparency. For an exactly fittable change leaves the same first residuals, so that we make the same transfers, transfers that do not alter the change in fit. Fuller treatment by midmean polishing is only one of very many processes that can follow elementary analysis by means without disturbing the transparency the elementary analysis gained.

If midmean polishing is to deal with large things, not just small ones, we must be careful and add recalculation of the general typical to center the residuals. Its stages will now be:

- find midmeans of each column of initial residuals,
- find midmeans of each row of initial residuals,
- subtract "column midmean plus row midmean" from each residual, find and subtract midmean of results, to give new residuals,
- add column midmeans to initial column adjustments, find and subtract midmean of results to give new column adjustments,
- add row midmeans to initial row adjustments, find and subtract midmean of results to give new row adjustments.

Table 24 shows the essentials of the computation for an illustrative example. Clearly the additional steps give us, in this case, just what we wanted.

table 24 of chapter 5

Crude and Careful Midmean Polishing Illustrated

a) DATA AND ITS STRIPE MIDMEANS

	1	2	3	4	
A	-4	-4	-4	12	(-4)
B	1	1	1	-3	(1)
C	1	1	1	-3	(1)
D	1	1	1	-3	(1)
E	1	1	1	-3	(1)
	(1)	(1)	(1)	(-3)	

b) RESULTS OF SUBTRACTING STRIPE MIDMEANS

A	-1	-1	-1	19	-5
B	-1	-1	-1	-1	0
C	-1	-1	-1	-1	0
D	-1	-1	-1	-1	0
E	-1	-1	-1	-1	0
	0	0	0	-4	2

Example: -1 = (-4) - (-4) - (-1).

Note: Stripe midmeans have been adjusted to give stripe adjustments.

c) RESULTS OF FURTHER SUBTRACTING GENERAL MIDMEAN

A	0	0	0	20	-5
B	0	0	0	0	0
C	0	0	0	0	0
D	0	0	0	0	0
E	0	0	0	0	0
	0	0	0	-4	1

Fuller treatment by midmean polishing gives substantial resistance, not as much perhaps, as we would like, but far more than elementary analysis by means alone. Consider the example of table 18 where we considered a large change in the value of one entry, and gave both before and after elementary analyses by means. Table 25 shows the effect, for this example, of midmean polishing the elementary analyses using means. As compared to the unpolished elementary analyses using means in table 18 we have made great progress. Indeed, as panel (d) shows, the resistance of the overall process is not too much worse than that of elementary analysis by midmean differences with midmean polishing. The approach works.

Once we know that this procedure works, there are two things we can do to make it work better or easier (one of each). The purpose of the first step -- analysis by means -- is to gain transparency. Usually we do not need such transparency to be exact. When approximate transparency will do, we could alter the initial table to eliminate obviously way-out values, analyze by means to get stripe adjustments and general typical, and then subtract these from the original data -- not the altered data -- to obtain the residuals that are to be midmean polished. Similarly, if the data is almost -- but not quite complete -- we can look hard at the table, insert reasonable values in place of the missing ones, do the mean analysis, find the stripe adjustments and general typical, subtract them from the original data -- leaving the holes open -- and then do the midmean polishing. (A few holes will not

table 25 of chapter 5

Midmean Polish Applied to the Example of table 18

Ends	Bobbin 1	2	3	4	
a) MIDMEAN POLISH OF E-A BY MEANS - ORIGINAL DATA					
A	.36	-.47	.05	.03	-2.18
B	+.02	.15	.07	-.31	.00
C	-.29	.24	-.25	.32	.46
D	-.35	.24	-.06	.13	.32
E	.33	-.96	.27	-.21	-.30
	-1.09	.14	-.13	.16	41.33
b) MIDMEAN POLISH OF E-A BY MEANS - MODIFIED DATA					
A	.28	-.86	-.30	19.47	-1.89
B	.00	-.08	-.12	-.70	.11
C	.00	.32	-.13	+.23	.28
D	-.12	.27	.00	-.01	.19
E	-.60	-.89	.38	-.30	-.47
	-1.28	.16	-.15	.34	41.44
c) DIFFERENCE IN POLISHED ELEMENTARY ANALYSES, COMPARE WITH TABLE 19					
A	-.08	-.39	-.35	19.44	+.29
B	-.02	-.23	-.19	-.39	+.11
C	+.29	+.08	+.12	-.09	-.18
D	+.23	+.02	+.06	-.14	-.13
E	-.08	+.07	+.11	-.09	-.17
	-.19	+.02	-.02	+.18	+.11

(Cont'd)

table 25 (Cont'd) - 2

d) STEM-AND-LEAF DISPLAYS OF CHANGES IN RESIDUALS PRODUCED BY THE SINGLE LARGE CHANGE

	(*) table 19(b)	(**) (c) above	(***) table 19(c)
more	x	x	xxx xxx xxx xxx x
+.4			
+.3			
+.2	94		
+.1	5595		
+.0	33723	372531153	
-.0	22	151230	
-.1	1139		
-.2	33	4052	
-.3			
-.4			
less			xxx xxx x

(*) from midmean differences with midmean polishing.

(**) from means with midmean polishing.

(***) from means above.

inconvenience the midmean polishing.) Though they may seem to be two different techniques, both alteration and filling-in are instances of the temporary use of supplied data. We count this as one way to do better in using means with midmean polishing.

The easier scheme only works when we have a complete table. Let us look at what the full process does, from the point of view of column adjustments. The steps could be taken in this order:

- find and subtract row means.
- then find and subtract column means.
- then find and subtract column midmeans.
- add together column means and column midmeans, and midmean adjust the sum to give final column adjustments.
- go back before the third step and find and subtract row midmeans.
- calculate row adjustments and a general typical.

This order is not quite the natural one, but it is a possible one. Its value to us is that the three steps associated with column adjustments are together in tight grouping. Finding column midmeans is transparent to changes constant by column. Thus we need not take out column means, but can replace the three steps by:

- find column midmeans and midmean adjust to give final column adjustments.

We can do the same thing for rows, except that we now must take out column means -- the means for the other stripes -- before we form row midmeans.

The overall process can look like this:

- find and subtract row means.
- find column midmeans of the result, midmean adjust to give final column adjustments.
- start anew, finding and subtracting column means.
- find row midmeans of these results, midmean adjust to give final row adjustments.
- start yet again, subtracting both row and column adjustments.
- find and subtract midmean of this set of results, thus getting final general typical and final residuals.

Table 26 illustrates this on our standard example. There are a few changes in the last decimal -- to be expected when we do the arithmetic in different orders without guard digits -- but we reach essentially the same answers. The streamlined form, with its convenient checking, has real advantages.

5I. <u>How far have we come</u>?

In this chapter we have come far and fast. Our progress has come from understanding and exploiting a half-dozen ideas:

- changing two things at once in various combinations.
- elementary analysis of a two-way table -- what it is, how to report it, various ways to do it.
- the workings and properties of simple fitting processes.
- the importance of differencing within a stripe.
- the parallel-line plot.
- coding residuals as a surprisingly powerful tool.

table 26 of chapter 5

The Streamlined Computation of An Elementary Analysis By Means With Midmean Polish

a) DATA AND INITIAL STEPS

	1	2	3	4	Sum	Mean		1	2	3	4	(5)
A	38.43	38.83	39.08	39.35	155.69	38.92		-.49	-.09	.16	.43	✓
B	40.27	41.63	211.28	41.19	164.37	41.09		-.82	.54	.19	.10	✓
C	40.42	42.18	41.42	42.28	166.30	41.58		-1.16	.60	-.16	.70	✓
D	40.22	42.03	41.47	41.95	165.67	41.42		-1.20	.61	.05	.53	✓
E	40.27	40.22	41.18	40.99	162.66	40.66		-.39	-.44	.52	.33	✓
Sum	199.61	204.89	204.43	205.76			(*)	-.82	.42	.14	.43	(.28)
Mean	39.92	40.98	40.89	41.15			(**)	-1.10	.14	-.14	.15	
					(*)	(**)						
A	-1.49	-2.15	-1.81	-1.80	-1.81	-2.18		-3.28	-2.04	-2.32	-2.03	
B	.35	.65	.39	.04	.37	.01		-1.09	.15	-.13	.16	
C	.50	1.20	.53	1.13	.83	.47		-.63	.61	.33	.62	
D	.30	1.05	.58	.80	.69	.33		-.77	.47	.19	.48	
E	.35	-.76	.21	-.16	+.02	-.34		-1.44	-.20	-.48	-.19	
(s)	✓	✓	✓	✓	(.36)							

Notes. Data to NW, row means subtracted to NE, (*) indicates midmeans, (**) indicates midmean adjusted midmeans, (s) indicates checks that sum to zero (almost), ( ) encloses midmean of midmeans. Column means subtracted to SW, and similarly treated. SE corner contains sums of row and column adjustments ready for use at next step.

(Cont'd)

table 26 (Cont'd) - 2

b) DATA STRIPE-ADJUSTED, WITH STEM-AND-LEAF DISPLAY

A	41.71	40.87	41.40	41.38
B	41.39	41.48	41.41	41.03
C	41.05	41.57	41.09	41.66
D	41.71	40.42	41.66	41.18

higher	10766
41.5	76
41.4	8017
41.3	98
41.2	8
41.0	593
lower	972

midmean = 41.36

c) ELEMENTARY ANALYSIS

A	.35	-.50	.04	.02	-2.18
B	+.03	.12	.05	-.33	.01
C	-.31	.21	-.27	.30	.47
D	-.37	.20	-.08	.11	.33
E	.35	-.94	.30	-.18	-.34
	-1.10	.14	-.14	.15	41.36

The key idea is clearly the gathering of data which can -- at one and the same time -- tell us about the differences among each of two sets of treatments or candidates. The great value of such data will become clearer and clearer as we proceed. There are very few -- if any -- situations in science -- and still few in technology -- where it pays us to gather repeated measurements on each of one set of candidates or treatments without bringing in a second set in such a way that there are but few complete repetitions of the circumstances of a measurement. Equally, nothing increases the power and insightfulness of our data analysis as much as to have two-way data with a strong added classification rather than just one-way data. This will gradually become clear.

The elementary analysis of a two-way table, the fitting of a partial description that can be written as a SUM of something CONSTANT BY ROWS and something CONSTANT BY COLUMNS, is an equally central idea. The appearance of a plus sign between "constant by rows" and "constant by columns" is NOT an accident, NOT something easily replaced by some other means of combination. We shall slowly learn a few of the reasons why this is so, some of which are built into us by our culture.

We have learned that a reasonably adequate report on an elementary analysis covers both partial description and residuals, uses both tables of numbers and displays. A good standard consists of:

- a table showing -- next to one another in the standard pattern -- residuals, row adjustments, column adjustments, general typical.

- a parallel-line display showing the partial description.

- two pairs of tables of coded residuals, in the standard pattern -- one pair in the order natural for rows and columns (can be omitted if neither has a natural order), one pair in the order determined by the row adjustments and column adjustments. (Sometimes these two will be the same, sometimes we can use tables with numerical residuals instead.)

As we will see later, even if we give this detailed a report, an elementary analysis is often only the first step in analyzing a two-way table.

We have learned a variety of ways to make an elementary analysis, some more resistant, others with less arithmetic. Each of them can be done in steps, first either rows and columns being treated, then the other.

If our table of data is incomplete, our elementary analysis will be based on summarized differences and a square combining table. The same considerations of choice apply to summarizing with mean, median or midmean as applied when we were summarizing a single batch with a single typical value. Again midmeans are almost always a good choice. Any combining table can be followed with the fuller treatment process, but sets of data that need it are in the minority.

If our table of data is complete, we can consider reducing our arithmetic by:

- analyzing in terms of means, and taking our chances with the mean's lack of resistance.

- using means and continuing by midmean polishing.

- using the compromise method -- especially if we have a long, narrow table -- where we analyze in one direction using midmean differences, form adjusted values, and use midmeans of stripes in the other direction.

While confining our discussion to our case of elementary analysis of a two-way table, we have actually learned a lot about what properties it is reasonable to require of the fitting process in simple fitting situations. We looked at a variety of ways of formulating these properties. They are perhaps most simply put in terms of this idea:

- What effect on fit comes from a change in data that is itself a possible fit (in more general situations a possible change in fit)?

In simple situations we can ask of our fitting process that it be transparent , in the sense that such a change in data produces exactly the same change in fit. If in addition, the fit is perfect whenever it can be, we have the standard collection of properties, with both useful and handy processes.

When we break the fitting process down, and think separately about row adjustments and about column adjustments, we find it useful to think about avoidance as well as about transparency. Perhaps the most important characteristic of a process of fitting row adjustments is that its result is unaffected by any change in the data that is constant by columns. And vice versa.

All the processes of elementary analysis we have considered gain this avoidance in the same way, sometimes openly and sometimes secretly. All our processes use -- or can be rewritten to use -- differencing within a stripe as an essential element that guarantees avoidance.

The parallel-line plot looks simple. It is simple. But its pattern tells us clearly -- if we will listen -- about the nature of the partial description of elementary analysis, both our elementary analysis and the general elementary analysis. We ought not to undervalue it.

Coded residuals are a tool that is much more powerful than we would expect. It would be natural to begin by supposing that some simple plot of residuals would tell us what we want to know. At least until a good new idea is found, this is not the case. Plots do not get the message across adequately. Coded residuals get it across surprisingly well. Making two pairs of (one in lag order, the other in adjustment order) tables of coded residuals takes some effort -- but probably little more than that needed to make a single, far-less-effective plot of residuals.

The symbolic scale that we use for coded residuals involves 10 tags, two for each of the five pieces into which we were already implicitly dividing the values of a batch:

- high wing, far out.
- high wing, not far out.
- body.
- low wing, not far out.
- low wing, far out.

The effectiveness of this simple coding scheme is repeatedly a pleasant surprise.

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5K. How to draw parallel lines

Our standard device for representing the partial description fitted in elementary analysis calls for two families of parallel lines. If the device is to be used as much as it should be, we must find it easy to draw parallel lines.

Two different solutions are worth knowing, one for "no tools" the other for "few tools". When we have no tools -- but do have graph paper -- we can always choose the slope of our lines as a simple ratio -- probably 1 to 1, 2 to 3, or 1 to 2. If we do this, constructing a line through a given point with this slope is rather easy. Finding a second point on the line by making round-numbered steps right-or-left and up does the trick.

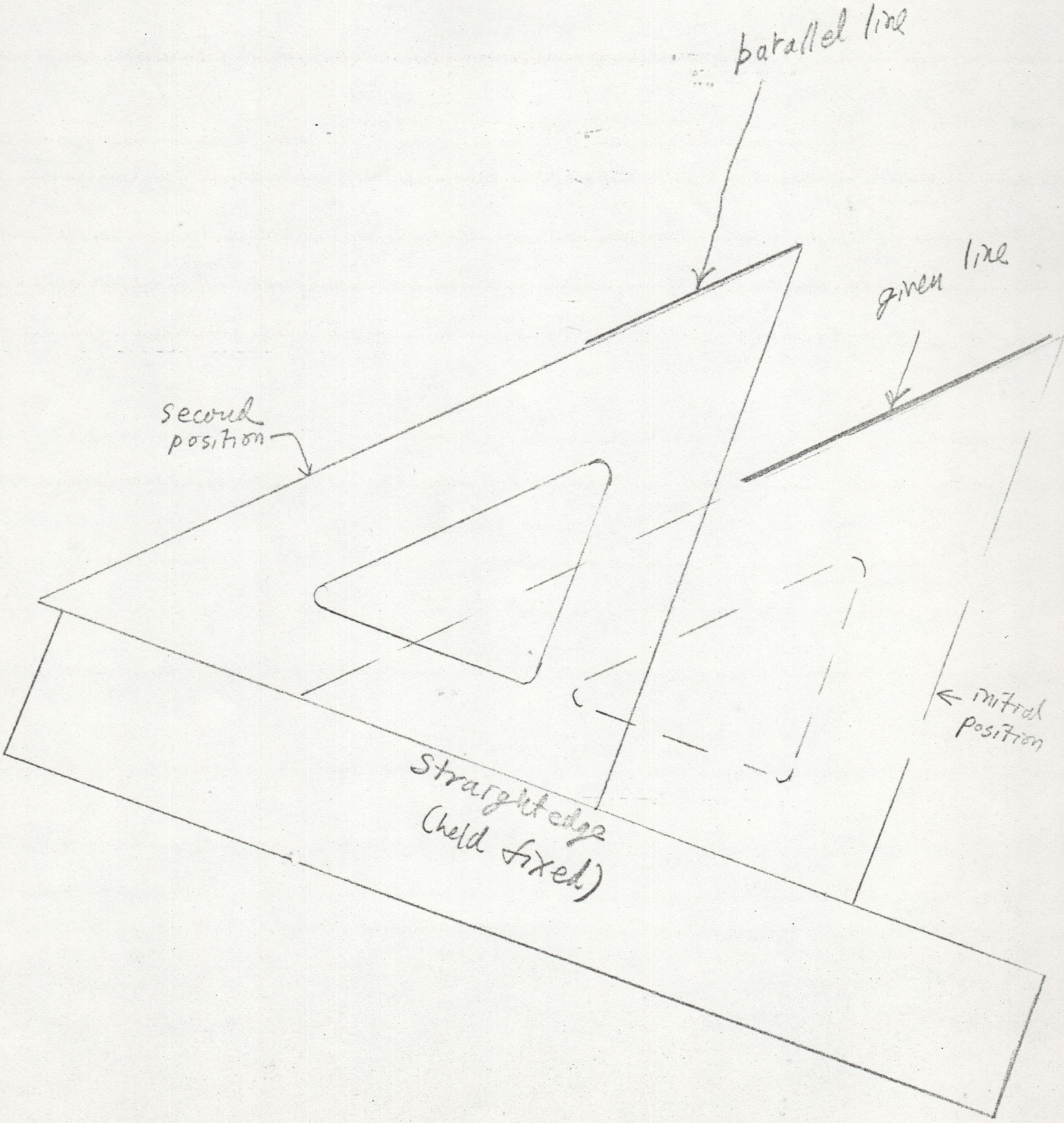
If we have a few tools, specifically a transparent triangle and a ruler, we can proceed more freely and even more easily. The basic construction for parallel lines is slipping the triangle along the straight edge, as shown in picture 27.

5L. How to find all differences

In this chapter, the taking of all differences among a family of treatments or candidates began to be a real effort. Even table 2, where there are 10 comparisons among pairs from A, B, C, D, E involves a noticeable amount of differencing. Any way of easing the burden deserves consideration. (Putting it all on the computer is a good solution, but one not always available.)

picture 27 of chapter 5

Drawing a parallel line



For those of us who add easier than we subtract, a trivial observation can speed things up appreciably when we are dealing with a complete two-way table. Clearly

A minus C = (A minus B) + (B minus C)

for the individual differences. If then we calculate the columns of differences for A-B, B-C, C-D, and D-E, we can find all the others by addition. Table 28 gives a detailed example. Similar principles can be applied to any number of candidates or treatments.

table 28 of chapter 5

Getting All Differences With Fewer Subtractions Illustrated on the Data and Calculations of Table 2

Bobbin	A-B	B-C	C-D	D-E	* A-C	* B-D	* C-E	** A-D	*** B-E	A-E
1	-1.84	-.15	.20	-.05	-1.99	.05	.15	-1.79	.00	-1.84
2	-2.80	-.55	.13	1.83	-3.35	-.42	1.96	-3.22	1.41	-1.39
3	-2.20	-.14	-.05	.29	-2.34	-.19	.24	-2.39	.10	-2.10
4	-1.84	-1.09	.33	.96	-2.93	-.76	1.29	-2.60	.20	-1.64

*formed by adding two adjacent columns to left, as in -1.99 = (-1.84) + (-.15), .05 = (-.15) + (.20), .15 = (.20) + (-.05).

**formed by adding C-D and A-C, as in -1.79 = (.20) + (-1.99).

***formed by adding D-E and B-D, as in .00 = (-.05) + (.05).

A-E is formed by adding any of various pairs. Using D-E and A-D or A-B and B-E give the strongest check. In any event the A-E differences should be checked from the original values.

*****

5P. IEDA processor

5R. Adding two other views to the parallel-line display

Plots 7 to 10 illustrated some things to do and some things not to do in trying to display the fitted partial description of elementary analysis. One of the main lessons of section 5C was that -- at least until we have been much more ingenious -- it was not wise to try to show residuals in place on the parallel-line display. Stacking up the residuals in a convenient corner of the display proved to be all right, but not specially useful. It is worthwhile to ask what other information can be usefully added to such display:

- for a naive user?
- for a user who has become used to this sort of display?

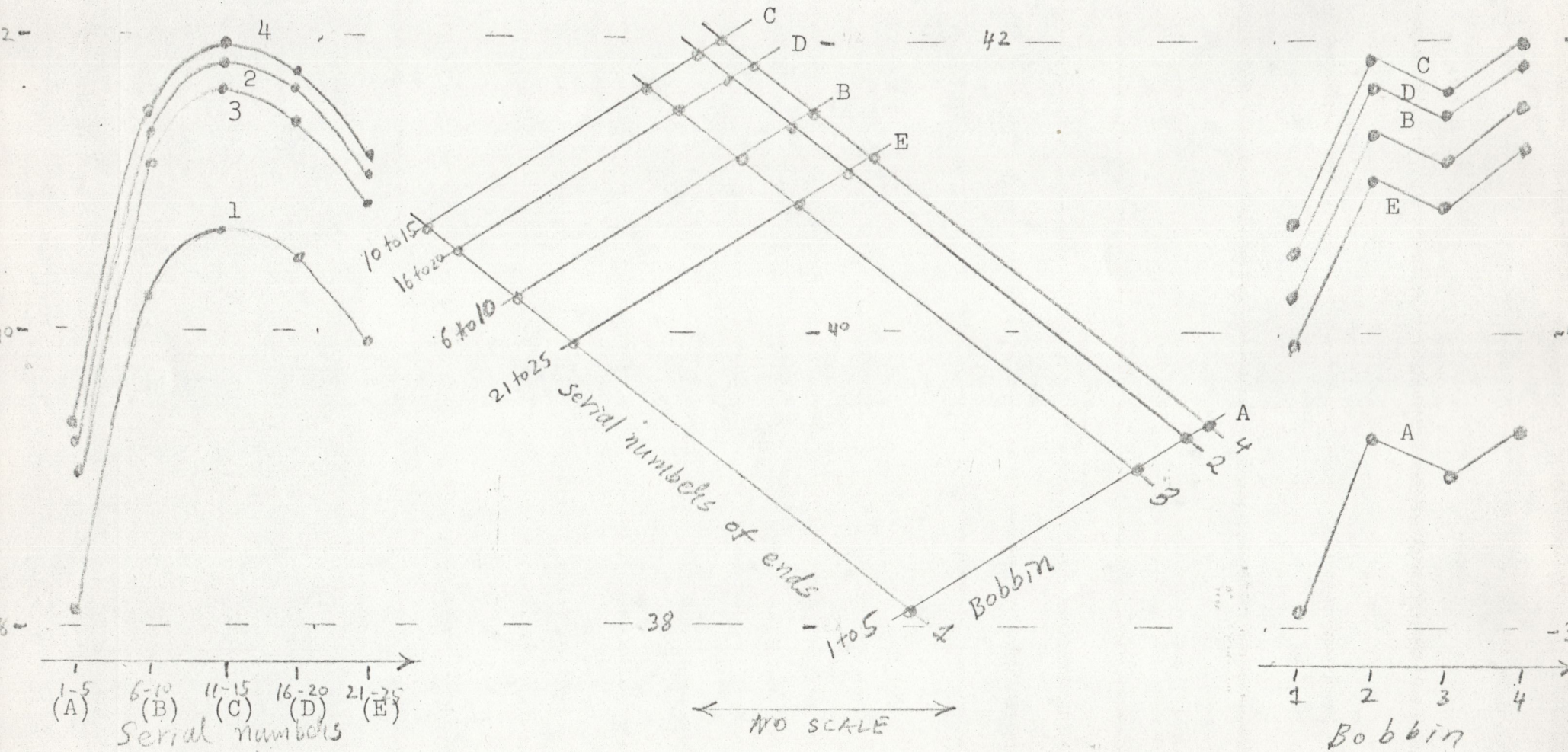
Plot 29 shows three things, side by side:

- in the center, our standard parallel-line display.
- to the left, using the same vertical scale, plots of fitted weight against end number (in groups) for each bobbin.
- to the right, still using the same vertical scale, plots of fitted weight against bobbin number for each end group.

For the viewer who does not mind looking at three things at once, this plot does quite well. Almost any question about the behavior of the fitted partial description can be answered by looking at some part of this plot.

In general, we need to be ready both for stripe labels that are ordered -- as these are -- and for those that have no inherent order. When stripe labels have no other

Three views of the fit to thread weight in terms of bobbin and end number
(Heights are weights of 5 pieces, each 95 yd long, in grams)



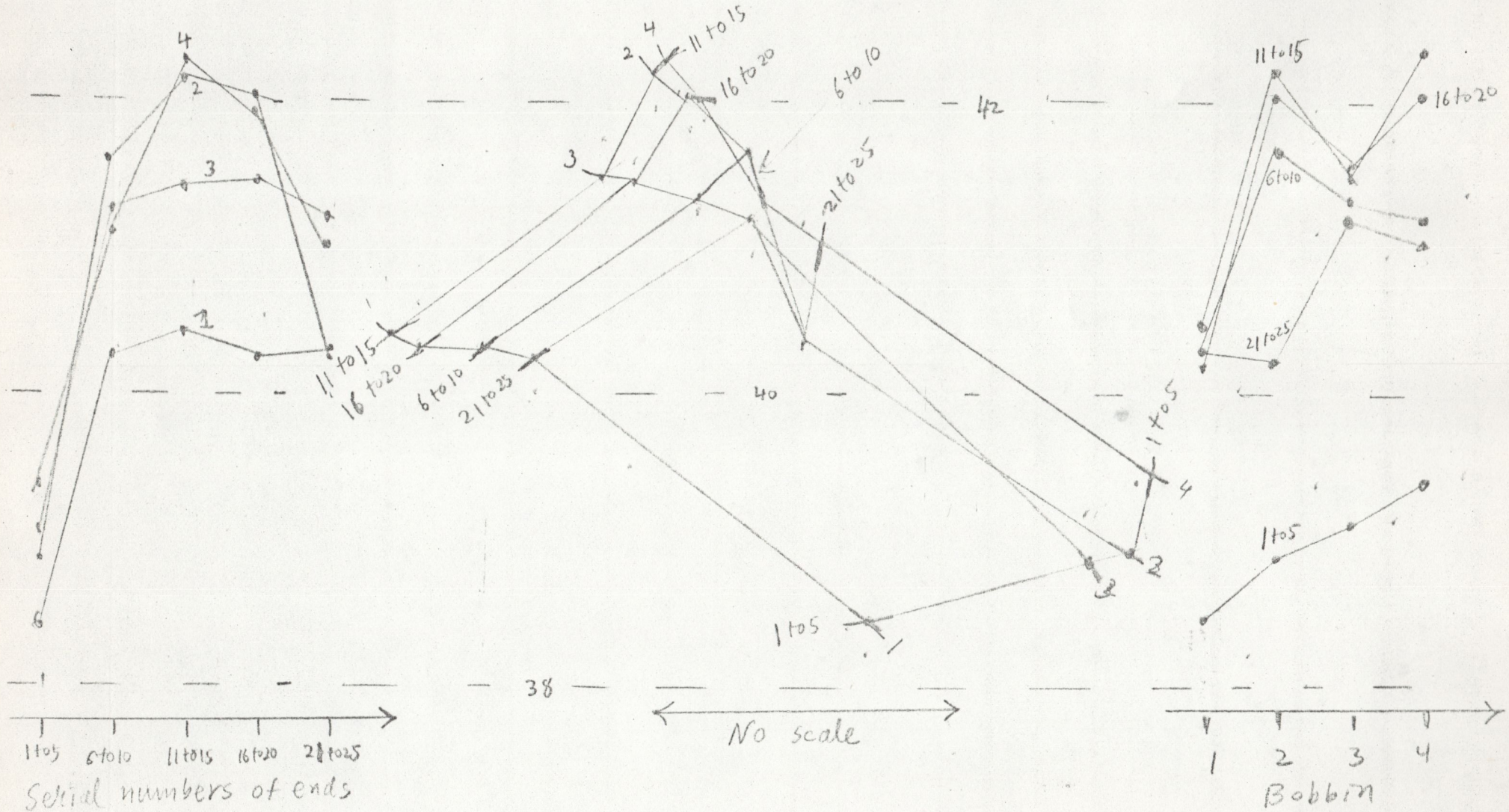
FITTED EFFECT OF END

FITTED COMBINED EFFECT

FITTED EFFECT OF BOBBIN

order, it is natural -- and recommended -- to use the order given by the fitted adjustments.

Having had so much success with such a plot for the partial description, it is natural to want to look at the same plot for the data. In doing this, we can gain, in the middle of the three views, by showing the data against an invisible background of the plot for the partial description. In plot 30, then, we have made the middle view by putting the observation points directly above or directly below the corresponding, invisible point for the partial description. A fair amount of information can be wrung from this plot, once we have seen and tried to set aside the main message:

- the observations are far more messy than the partial description.

Three views of the thread-weight observations
(Heights are weights of 5 pieces, each 25 yd long, in grams)